

November 2024 Problem of the Month

We define $f(n)$ as a function from the positive integers to the positive integers as follows.

$$f(n) = \begin{cases} 3n + 1 & n \text{ is odd} \\ \frac{n}{2} & n \text{ is even} \end{cases}$$

Let $f^k(n)$ be the function iterated k times.

If we start at 3, we get the sequence 3, 10, 5, 16, 8, 4, 2, 1. It took 7 steps to get to 1 so we can say $f^7(3) = 1$. We can also see $f^5(3) = 4$.

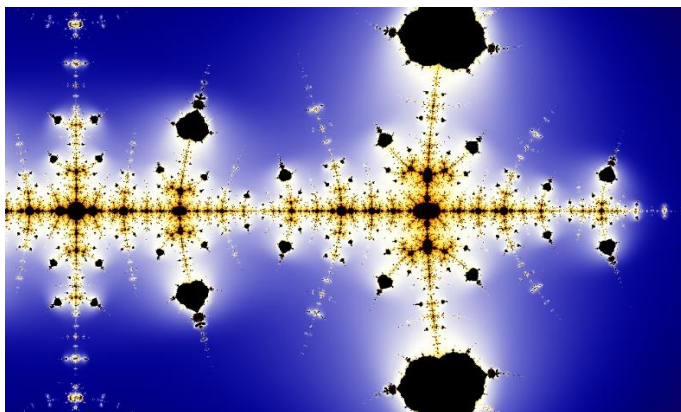
a) Find all positive integers n which satisfy the following.

$$f^3(n) = n - 2$$

b) Find all positive integers n which satisfy the following.

$$f^5(n) = n + 4$$

Please email solutions to Dr London at slondon@luc.edu in PDF form by 11:59 pm on November 30. Please clearly state your name, whether you are an undergraduate, and your major on your solution. The solution with the best explanation from a Loyola undergraduate will be the winner. Other correct solutions or solutions with enough progress may be recognized too.



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